Subject Code:24BM11RC02 R-24 Reg No: [ ]

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN

(AUTONOMOUS)

(Affiliated to Andhra University, Visakhapatnam)

I B.Tech. - II Semester Regular Examinations, June / July – 2025

LINEAR ALGEBRA AND VECTOR CALCULUS

(Common to All Branches)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

UNIT-I

1. a. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ by reducing to echelon form. [7]

   b. Find the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ has infinite number of solutions. [7]

OR

2. a. Apply Gauss elimination method to solve the equations $x + 4y - z = -5, x + y - 6z = 12, 3x - y - z = 4.$ [7]

   b. Apply factorization method to solve the equations $3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7.$ [7]

UNIT-II

3. a. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and find its inverse. [7]

   b. Find the singular value decomposition of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ [7]

OR

4. Find the eigen values and eigen vectors of the matrix A and A^{-1} where $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. [14]

UNIT-III

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ into the canonical form by an orthogonal reduction and find its nature. [14]

OR

6. Reduce the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ to the diagonal form and find A^4 [14]

UNIT-IV

7. a. Find the directional derivative of $f(x,y,z) = x^2 - y^2 + 2z^2$ at $P(1,2,3)$ in the direction of the vector PQ, where Q is the point (5,0,4). [7]

b. Prove that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. [7]

OR

8. a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. [7]

b. Show that $\nabla.(f\overline{A}) = f(\nabla.\overline{A}) + \overline{A}.(\nabla f)$ where f is a scalar function and $\overline{A}$ is vector function. [7]

UNIT-V

9. a. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$. [7]

b. Using Green's theorem, evaluate $\oint_C (xy + y^2)dx + x^2\,dy$ Where C is bounded by $y = x$ and $y = x^2$ [7]

OR

10. a. Apply Gauss's divergence theorem to find $\iint_S F.N\,ds$ where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [7]

b. Using Stoke's theorem, evaluate $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$, where C is the boundary of the triangle with vertices (2,0,0),(0,3,0), (0,0,6). [7]